



NSW Department of Education

# Numeracy guide

## Kindergarten to Year 2

A guide to support conversations about  
evidence-based practice for leadership teams

Literacy and numeracy  
2024 update



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This document is designed for online use.

# Purpose of the resource

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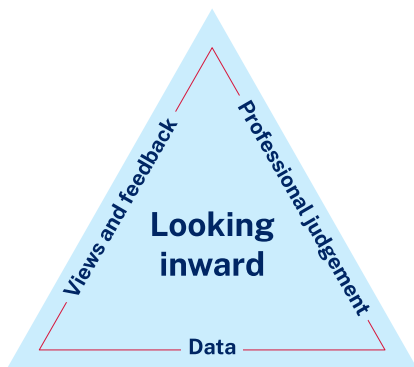
The purpose of this guide is to support directors, educational leadership, principals, school leadership teams and teachers to have informed conversations about evidence-based numeracy teaching across curriculum areas in primary school contexts.

This guide can:

- assist with an analysis of current practices
- help to inform planning for school improvement with numeracy
- suggest ways to build teacher capacity and understanding of numeracy with explicit classroom practices and professional learning resources.

## Situational analysis

This guide can be used as part of the situational analysis in the following ways:



**Looking inward** includes analysis of data such as evidence of staff knowledge and perceptions around numeracy, and evidence of students' current skills in numeracy.

The guide should be used in conjunction with a thorough analysis of internal and external measures such as:

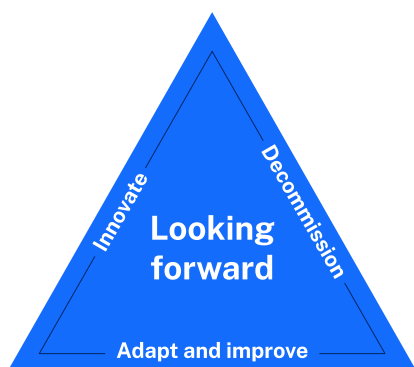
- School-based data
- [Interview for Student Reasoning \(IfSR\)](#)
- Best Start Kindergarten Assessment
- [National Numeracy Learning Progression](#)
- NAPLAN data
- Check-in assessment.



**Looking outward** includes comparing the school's approach on the teaching of numeracy to the research on effective teaching of numeracy.

This guide:

- explains the key aspects of numeracy
- describes evidence-based practices for effective teaching of numeracy
- describes the roles and responsibilities of all stakeholders.



**Looking forward** includes making decisions in response to information gained in the **Looking inward** and **Looking outward** phases.

This guide makes suggestions for ways to adapt and improve the teaching of numeracy across K-2.

# Introduction

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This resource incorporates research on the connection between mathematics and numeracy, and the centrality of the idea of working mathematically drawn from the K-10 Mathematics syllabus. It also highlights evidence-based practices to inform teaching and learning in number and place value, patterns and algebra, additive thinking, multiplicative thinking and proportional thinking.

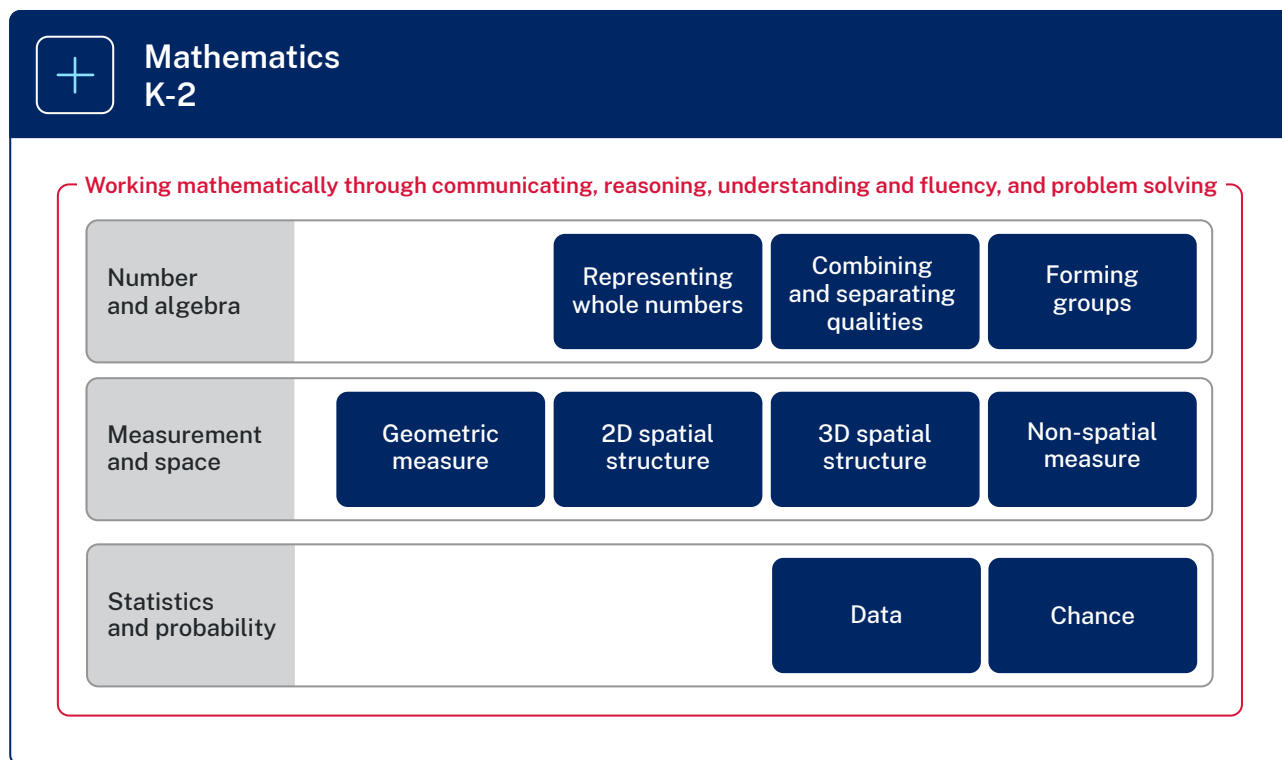
## Numeracy and mathematics

Numeracy and mathematics are so intricately linked that it is difficult to define one without reference to the other. Whilst they are interdependent, the relationship between mathematics and numeracy changes as children progress through schooling.

Numeracy is considered as the confident application of mathematical skills, understanding and dispositions across areas of learning and within our daily lives. It involves recognising where mathematics can be used and being able to select the relevant mathematical tools and make sense of the solutions. As described by the NSW Mathematics K-10 syllabus (2022), 'By studying mathematics, students develop essential numeracy skills and fluency, while nurturing the ability to think logically, critically and creatively. They learn about patterns and reason about relationships, creating opportunities to generalise their solutions and to solve non-routine problems'. Mathematics provides the building blocks for the dispositions, understandings and skills needed for children to become numerate before extending out into developing deeper and richer understanding and more nuanced skills. The [Every Student Podcast](#) with Michelle Tregoning highlights the power and relevance of mathematics. Mathematics is necessary for numeracy, but numeracy is not all of mathematics. Both mathematics and numeracy are foundational for success in everyday life.

The distinction between numeracy and mathematics supports teachers to identify opportunities to assist students to become numerate as they develop the knowledge and skills to use mathematics confidently across learning areas at school and in their lives more broadly.

## Working mathematically



## Working mathematically in mathematics K-2

The working mathematically processes in the NSW Mathematics syllabus are:

- communicating
- understanding and fluency
- reasoning
- problem-solving.

Students learn to work mathematically by using these processes in an interconnected way. The coordinated development of these processes results in students becoming mathematically proficient.

When students are working mathematically it is important to help them to reflect on how they have used their thinking to solve problems. This assists students to develop mathematical habits of mind.<sup>1</sup>

Students need many experiences that require them to relate their knowledge to the vocabulary and conceptual frameworks of mathematics.

<sup>1</sup> Cuoco A, Goldenberg EP and Mark J (2010) 'Contemporary Curriculum Issues: Organizing a Curriculum around Mathematical Habits of Mind', *The Mathematics Teacher* MT, 103(9):682-688, doi:10.5951/MT.103.9.0682.

## Overarching working mathematically outcome

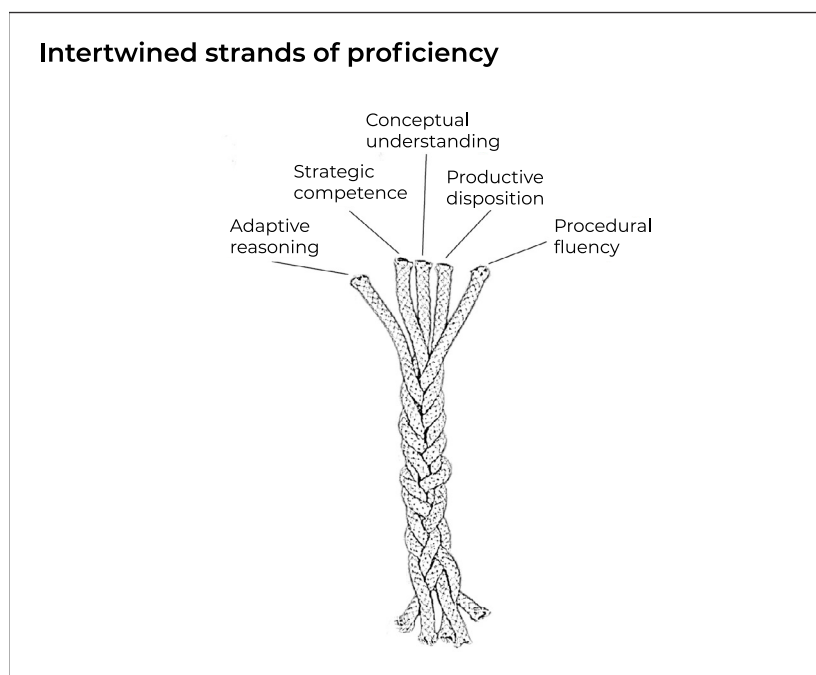
To highlight how these processes are interrelated, in Mathematics K-2 there is one overarching working mathematically outcome:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

The working mathematically outcome describes the thinking and doing of mathematics. In doing so, the outcome indicates the breadth of mathematical actions teachers need to emphasise.

In alignment with these Working Mathematically components, Kilpatrick, Swafford and Findell (2001) discuss the notion of ‘mathematical proficiency’ as a way to capture the necessary skills, knowledge and understandings a learner requires to learn mathematics successfully. They highlight 5 strands or components that are interwoven and interdependent in the development of proficiency in mathematics:

- conceptual understanding – comprehension of mathematical concepts, operations, and relations
- procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence – ability to formulate, represent, and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation, and justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.



The Working Mathematically components and the five strands of mathematical proficiency are foregrounded in language use as students explore and develop their numeracy skills. Teachers need to be aware that students with differing English language proficiency may display different ways of working and thinking mathematically, and plan for explicit teaching for the language demands of numeracy.

For further information on the strands of mathematical proficiency see: Kilpatrick, Swafford and Findell (2001) *Adding It Up: Helping Children learn mathematics*, (Chapter 4: The strands of mathematical proficiency, p.116) National Research Council. 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: The National Academies Press.

## Learning across the curriculum (new K-2 Mathematics)

### Numeracy

Numeracy involves understanding and applying mathematical knowledge and skills in a wide range of contexts. The application of mathematics across the curriculum enriches the study of other learning areas and helps to develop a broader and deeper understanding of numeracy.

A 'numerate person' (Hogan, 2000) uses a blend of:

- mathematical knowledge: concepts and skills within mathematics
- contextual knowledge: to recognise and link mathematics to broader situations
- strategic knowledge: to apply mathematics in situations and evaluate if the solution is reasonable.

Students are provided with opportunities to:

- develop knowledge and skills to use mathematics confidently at the school level and beyond
- develop the mathematical proficiencies of understanding, fluency, reasoning and problem-solving
- apply their knowledge of mathematics in a variety of contexts and circumstances, choosing the appropriate mathematical concepts, and critically evaluating its use.

## Defining numeracy

Numeracy is complex and multifaceted. While the evidence base for understanding and teaching numeracy is largely shared with mathematics, some researchers have attempted to define numeracy and describe its components.

To support teacher reflection and planning to inform effective numeracy instruction, Goos, Geiger and Dole (2012) created a model for 21st century numeracy that outlines five main elements:

Element of model	Description of element
<b>Mathematical knowledge</b>	Mathematical concepts and skills; problem solving strategies; estimation capacities.
<b>Contexts</b>	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings.
<b>Dispositions</b>	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
<b>Tools</b>	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking.
<b>Critical orientation</b>	Use of mathematical information to make decisions and judgments; reason and support arguments.

The model supports teacher understanding of the complex interplay of components that contribute to student numeracy development, as well as the need to provide learning experiences that specifically target each element.



## Building numeracy through effective mathematics teaching practices

It is generally accepted that becoming a confident, creative user and communicator of mathematics, particularly in the primary years of schooling, is foundational for numeracy. To support teachers to develop these foundations and nurture student development of 21st century numeracy, teachers and leaders need to enact effective teaching practices. The teacher plays a pivotal role, incorporating a mix of pedagogies ‘including play based and structured activities’ which provide students with high cognitive challenge and achievement’ (Queensland Government, 2020, p13). An understanding of the continuity of learning as students begin school acknowledges prior mathematical skills and supports informed planning to build upon existing knowledge. In **Principle to Actions**, the National Council for the Teaching of Mathematics (NCTM, 2014), sets forth a set of research-informed teaching actions. These eight practices ‘provide a framework for strengthening the teaching and learning of mathematics... [and] represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics’ (NCTM, 2014, p.9).

### Mathematics teaching practices

#### Establish mathematics goals to focus learning

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

#### Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

#### Use and connect mathematical representations

Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics of mathematics concepts and procedures and as tools for problem solving.

#### Facilitate meaningful mathematical discourse

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analysing and comparing student approaches and arguments.

#### Pose purposeful questions

Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.

## Mathematics teaching practices

### Build procedural fluency from conceptual understanding

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so students, over time, become skilful in using procedures flexibly as they solve contextual and mathematical problems.

### Support productive struggle in learning mathematics

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

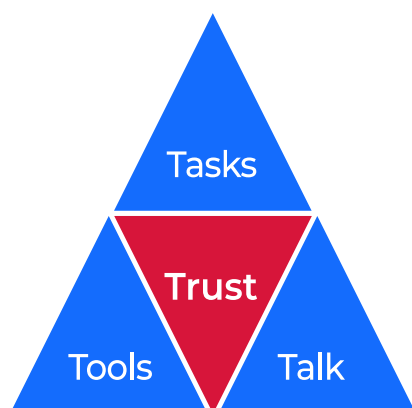
### Elicit and use evidence of student thinking

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

National Council for the Teaching of Mathematics (2014, p.10)

## The teaching tripod

In his book **Transforming Primary Mathematics**, Mike Askew (2016) discusses that teachers should plan for communities of learners using the teaching tripod and its practical considerations. The teaching tripod focuses on three areas: tasks, tools and talk. He suggests that by attending to each of the three areas, lessons can be designed to be open enabling students to bring their mathematical knowledge to the forefront whilst still allowing teachers to guide the mathematical content that emerges.



**Tasks** are at the centre of teaching and learning. Tasks need to focus on developing students' active sense making, providing opportunities to construct knowledge inside of the learning whilst making connections to real life experiences. Mike Askew (2016) suggests this requires teachers to shift the structure of their lessons from demonstrations at the beginning of the lesson to allowing students to work on open-ended tasks with a certain amount of uncertainty so they 'engage mindfully and bring their sense making to the activity' (p.127).

The use of **tools** supports students' mathematical development. Tools can be physical manipulatives and can also include models that teachers introduce, such as open number lines or arrays, to scaffold the abstract nature of mathematics. Through the use of manipulatives and models, and in the presence of reasoning and discussion, the underlying mathematics can be made visible to students and they are able to 'see' and make sense of what is actually happening. Mike Askew (2016) suggests base ten blocks, interlocking cubes, 2D shapes and 3D objects, paper and the intentional use of colour as highly effective tools. Once students have a sound foundation in using tools for thinking about a particular concept, they can deal with the concept in more formal and abstract ways.

**Talk** is central to mathematics lessons. Students need to be encouraged to talk mathematics, not simply talk about mathematics which means that mathematical vocabulary becomes part of the classroom discourse. Mike Askew (2016) discusses three aspects of talk that aim to support mathematical thinking and these include:

- the importance of listening (as well as speaking) – teachers and students need to practice close and deep listening of each other's ideas. Students should be encouraged to listen attentively and build on each other's ideas as they contribute to discussions.
- recognising the difference between discussion and dialogue – where discussions are about establishing and defending an idea or view, dialogue is about the exchange of ideas and views. In mathematics, both are needed.
- focusing on mathematical reasoning – Mike Askew (2016) asserts that reasoning should not only happen after an answer has been obtained, and a student is explaining and justifying their solution, but happens as the solution is being found.

Teachers need to consider the needs of EAL/D students when planning for talk and the types of scaffolds that will support students to talk mathematically, which may include think, pair, share, word banks, sentence starters and anchor charts.

Central to Mike Askew's (2016) tripod of tasks, tools and talk is a fourth 'T' – **trust**. He explains:

If we want children to engage with mindful, meaningful mathematical tasks then we, teachers, have to trust that they will come up with improvised solutions that can be collectively crafted into the canonical mathematics. They, the children, have to trust that we are genuinely interested in their thinking. And we have to trust ourselves to be able to make sense of what the children produce. (p.157)

By developing trusting classroom communities, teachers are working towards creating positive attitudes where students are able to take risks, think, reason, communicate, reflect and critique the mathematics they come across.

## National Numeracy Learning Progression

National learning progressions describe the skills, understandings and capabilities that students typically acquire as their proficiency increases in a particular aspect of the curriculum over time.

They describe the learning pathway(s) along which students typically progress in particular aspects of the curriculum regardless of age or year level, and are designed to help teachers ascertain the stage of learning reached, identify any gaps in skills and knowledge, and plan for the next step to progress learning (National Numeracy Learning Progression, Version 3 2020, p.5).

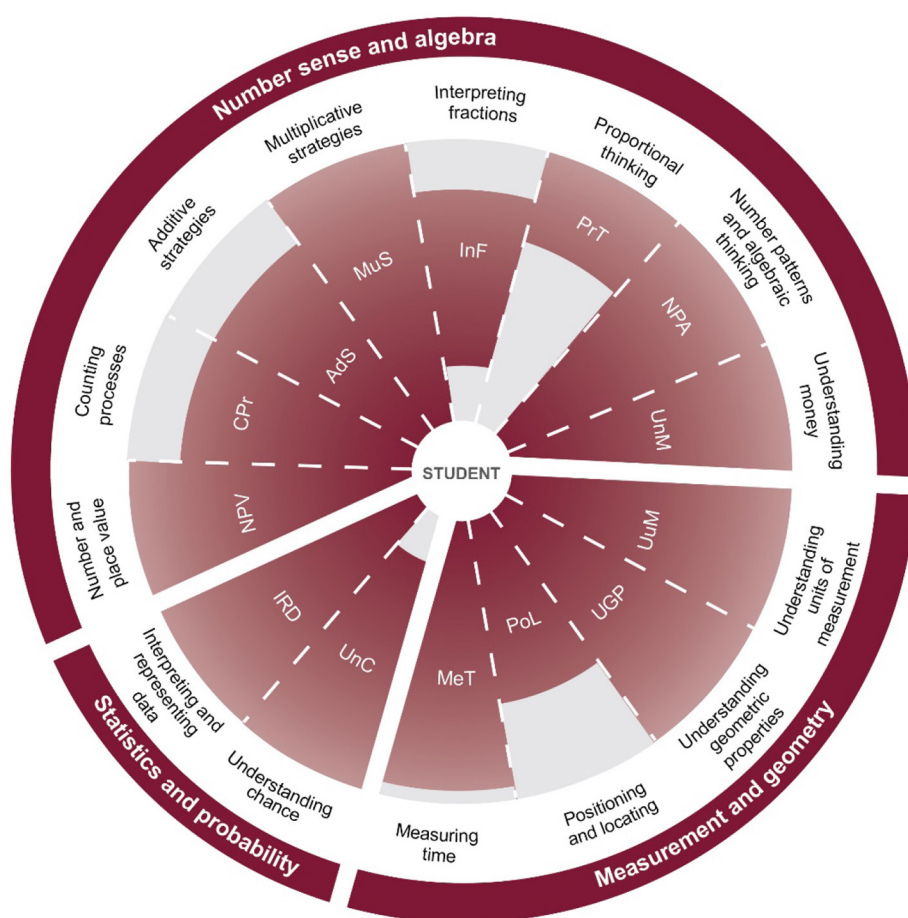


Figure 1. Elements and sub-elements of the National Numeracy Learning Progression

(ACARA, 2020, p.5)

### **Number sense and algebra**

- Number and place value
- Counting processes
- Additive strategies
- Multiplicative strategies
- Interpreting fractions
- Proportional thinking
- Number patterns and algebraic thinking
- Understanding money

### **Measurement and geometry**

- Understanding units of measurement
- Understanding geometric properties
- Positioning and locating
- Measuring time

### **Statistics and probability**

- Understanding chance
- Interpreting and representing data

# Leading to improve numeracy

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Principals and leadership teams have a pivotal role in driving numeracy improvement in schools. This guide is designed to support conversations about numeracy improvement in the context of school strategic planning.

## Principal

### To drive improvement in numeracy, a principal:

- ensures staff have a consistent understanding of evidence-based instruction and assessment of the numeracy demands within all key learning areas to embed effective teaching of numeracy
- builds numeracy leadership across the whole school, including the critical relationships across the curriculum
- promotes improvement as the shared responsibility of all staff in all key learning areas
- fosters an environment where a shared culture of improvement is valued and supported across the school creating positive mindsets and dispositions for all stakeholders, including the school community
- leads a coherent and shared school understanding of numeracy development using the relevant NSW syllabus outcomes in conjunction with the [National Numeracy Learning Progression](#)
- uses the themes of the updated 'What works best: 2020 update' as a starting point to establish a consistent approach to teaching and learning
- facilitates leadership, collaboration and collegial approaches to whole school improvement by clearly identified structures and processes embedded in the school's strategic improvement plan that allow opportunities for leaders and teachers to:
  - collaborate with teams
  - engage in professional learning
  - explore and respond to research and evidence
  - co-plan, co-teach and co-evaluate including peer observations and lesson studies
  - engage in classroom 'learning walks and talks'
- ensures a powerful and collaborative whole school approach to data analysis and student progress monitoring in order to measure impact to target future teaching and learning
- budgets for professional learning, the purchase of resources and intervention programs for students who need further support
- creates positive, challenging learning environments for all learners
- recognises the cultural diversity of school communities, facilitates staff to understand and acknowledge the numeracy demands of students from Aboriginal and EAL/D backgrounds

- ensures literacy and language learning is properly supported and emphasised, recognising these are fundamental to the development of student numeracy
- incorporates strategies within the school plan that comply with the [Aboriginal Education policy](#) and supports staff to participate in learning pathways to build competencies in Aboriginal cultures
- ensures school policies and practices are consistent with the [Multicultural Education policy](#) and for including multicultural education strategies in their school plans which provide teaching and learning programs that develop intercultural understanding, promote positive relationships and enable all students to participate as active Australian and global citizens
- drives a culture of inclusion and inclusive learning that benefits all students, including students with disability. Ensures reasonable adjustments are provided as required under the disability standards, so every student improves every year. For more information about adjustments visit the [Disability, learning and support webpage](#)
- leads the optimal talent development of high potential and gifted students across all domains of potential within supportive learning environments that develop the whole student. For more information visit the [High potential and gifted education webpage](#).

## Leadership team

### To drive improvement in numeracy, a leadership team:

- engages with current research and evidence regarding the development of numeracy and mathematics to support teacher knowledge and understandings
- supports teachers to recognise the cultural diversity of their students and builds their capacity to understand and acknowledge the numeracy learning needs of Aboriginal and/or Torres Strait Islander students and EAL/D learners ensuring quality teaching and assessment practices and resources are culturally inclusive. For more information visit the [Aboriginal education and communities webpage](#) for Aboriginal education in NSW schools and the [Multicultural education webpage](#) for EAL/D education
- supports teachers to build a culture of inclusive education for all students to achieve learning outcomes, supported by reasonable adjustments and teaching strategies tailored to meet their individual needs. For more information about adjustments visit the [Disability, learning and support webpage](#)
- supports teachers in the assessment and identification of the specific numeracy learning needs of high potential and gifted students across all domains of potential and effective differentiation for those students. For more information visit the [High potential and gifted education webpage](#)
- ensures school teams and teachers have given consideration to transition to kindergarten and the early identification of student numeracy needs

- reviews current teaching practices, teacher understanding and beliefs about numeracy and mathematics which can include:
  - systematic analysis of teaching using various models of observation and reflection
  - systematic analysis and refinement of teaching and learning programs and resources
  - clearly defined roles and responsibilities of staff to meet the needs of students
  - identifying, creating and implementing differentiated professional learning opportunities to build the capacity, knowledge, confidence and dispositions of teachers
  - ensuring current resources are supported by rigorous research evidence and enhance student learning and engagement
  - an understanding of the role of age-appropriate pedagogies in early years classrooms
- identifies opportunities for numeracy to be enhanced in all key learning areas, ensuring dispositions, understanding and skills are transferred across a variety of contexts through:
  - establishing a shared understanding of evidence-based practices for the targeted teaching of numeracy (see Goos et al's model on p.8)
  - collaboratively identifying and embedding numeracy into teaching and learning programs
  - developing a shared whole-school responsibility and consistent use of language
- develops a deep understanding of current NSW K-10 Syllabuses and the links to the [National Numeracy Learning Progression](#) (currently mapped to the Mathematics K-10 Syllabus) to:
  - identify students' prior knowledge, current understanding, skills, and interests
  - plan for targeted teaching
  - monitor individual student progress
  - support students to successfully engage with the numeracy demands of the syllabuses
- establishes systems and structures to:
  - ensure assessment of numeracy across curriculum areas is balanced and ongoing, including mathematics
  - build leadership of colleagues by working shoulder to shoulder with teachers to investigate, plan programs, with embedded effective teaching strategies and the development of quality assessment
  - use data to make informed decisions about targeted teaching and differentiate learning
  - lead, plan and support high quality teaching and learning programs that engages students through inquiry-based, challenging tasks



- engage in regular in-class support including classroom coaching, observation and feedback, learning walks and talks, co-teaching and the analysis of work samples to identify the impact of practice and programs on student outcomes and engagement
- participate in professional conversations to evaluate and modify teaching strategies and programs.

## Teacher

### To drive improvement in numeracy, a teacher:

- engages students in purposeful tasks and learning experiences that require deep thinking about important concepts and relationships
- creates an environment that encourages collaboration, educative risk-taking, meaningful talk, and uses students' misconceptions and errors as building blocks for learning
- provides play-based learning experiences that enable students to consolidate, practice, apply and transfer their numeracy skills
- designs opportunities for students to regularly experience productive struggle, exploring ideas and concepts to develop and use an increasingly sophisticated range of skills
- designs opportunities for students to practise what they are learning whether it be to improve fluency, problem-solving skills, or enrich conceptual understanding
- intentionally chooses and uses tasks because they meet a specific mathematical purpose, offering appropriate levels of challenge and opportunities for feedback for all students
- facilitates and plans productive classroom dialogue that encourages and supports students to justify their thinking and actions, drawing on a range of pedagogical practices and representational competencies
- supports students in connecting different strategies, approaches, representations, and concepts
- uses everyday experiences to design teaching and learning activities
- explores and uses multiple, meaningful representations to develop communicating skills, and understanding
- models how to work flexibly with numbers, operations and other critical ideas
- provides opportunities to use an ever-increasing range of representations
- builds on students' existing thinking through questioning and modifying tasks to provide alternative pathways to understanding
- plans learning experiences that enable students to build on their existing proficiencies, interests, confidence and experiences
- selects concrete materials/manipulatives that engages students in mathematical thinking to support them to represent mathematical ideas explicitly and concretely

- provides students with opportunities to work as a whole class, in small groups, in focus groups and on their own to make sense of and use ideas
- exemplifies the importance and real-world application of numeracy and mathematical competencies
- draws clear and consistent links between abstract and concrete mathematical concepts
- supports and scaffolds the background knowledge, cultural assumptions and language demands of numeracy to meet the needs of students from Aboriginal and/or Torres Strait Islander backgrounds. For more information visit the [Aboriginal education and communities webpage](#) for Aboriginal education in NSW schools
- designs targeted teaching and learning numeracy tasks that address the specific learning and wellbeing needs of students from culturally diverse backgrounds, including newly arrived and refugee students. For more information visit the [Multicultural education webpage](#) for EAL/D education
- makes reasonable adjustments according to a student's personalised learning and support needs to access assessment, syllabus outcomes and content on the same basis as their peers to meet the numeracy demands of the curriculum. For more information about adjustments visit the [Disability, learning and support webpage](#)
- applies numeracy evidence-based approaches that extend and challenge high potential and gifted students beyond their current level of mastery across all domains of potential. Develops, designs and teaches differentiated learning programs and provides experiences that meet the advanced learning needs of students. For more information visit the [High potential and gifted education webpage](#).

# Number sense and place value

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Number sense is considered a way of thinking about mathematical situations in order to make judgements, interpret data and communicate effectively (Booker, 2014). Number sense can be described as:

a person's general understanding of numbers and operations along with the ability and inclination to use this knowledge in flexible ways. Number sense is crucial for making mathematical judgements and developing useful strategies for handling numbers and operations (McIntosh et al. 1997, p.3).

As such, number sense 'requires a deep knowledge of numbers and operations that can be used confidently and flexibly in multiple contexts, the capacity to explain and justify one's thinking and generalise, and an appreciation of pattern and mathematical structure.' (Siemon, Warren, Beswick, Faragher, Miller, Horne, Jazby, Breed, Clark and Brady, 2020, p.265).

Place value is foundational to developing a deep sense of number as students learn to appreciate the base 10 numeration system and that the value represented by a digit in a number is based on its position in the number. It is also about understanding the significance that '10 of these is one of those' and '1000 of these is 1 of those' (Siemon et al., 2019).

Number sense develops over a long period of time and requires meaningful, challenging experiences focussed on a broad range of critical ideas including:

- seeing mathematics as something we make sense of and use to share ideas
- noticing patterns and relationships
- making sense of numbers 0-9
- making sense of 10 and beyond (including place value)
- making sense of fractions (including decimals and percentages)
- using and making connections between different representations
- making sense of operations
- thinking multiplicatively.

## Syllabus and progression links

Number sense underpins all aspects of the NSW Mathematics K-10 syllabus. Achievement of outcomes in number and algebra is dependent upon students having strong number sense.

NSW Syllabus	National Numeracy Learning Progression
<p>All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including <a href="#">Mathematics</a>, <a href="#">Science and technology K-6</a>, <a href="#">Science 7-10</a>, <a href="#">History</a>, <a href="#">Geography</a> and <a href="#">PDHPE</a>. Numeracy is also embedded within <a href="#">Creative arts</a>.</p> <p>Early Stage 1: 0, MAE-RWN-01, MAE-RWN-02</p> <p>Stage 1: MAO-WM-01, MA1-RWN-01, MA1-RWN-02</p> <p>For more information on syllabus connections, see the <a href="#">Making connections in Mathematics</a> document series on the NESA website.</p>	<p>Early Stage 1: NPV1–NPV4, CPr1–CPr5 and NPA1–NPA2</p> <p>Stage 1: NPV2–NPV6 and CPr5–CPr7</p>

## Further support

Professional learning	Assessment tools and resources
<ul style="list-style-type: none"> <li>• <a href="#">Becoming mathematicians: How numbers and fractions work</a> (Professional learning video with accompanying resource)</li> <li>• <a href="#">Becoming mathematicians: Quantifying collections</a> (Professional learning video with accompanying resource)</li> <li>• <a href="#">Improving reading and numeracy suite</a>:             <ul style="list-style-type: none"> <li>◦ Number and place value (primary)</li> </ul> </li> <li>• <a href="#">Quality curriculum implementation K-6</a> (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6)</li> <li>• <a href="#">Mathematics K-2 microlearning</a> (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))</li> </ul>	<ul style="list-style-type: none"> <li>• <a href="#">IfSR-Number and place value</a> (IfSR-NP) resources</li> <li>• IfSR-NP diagnostic online assessment – <a href="#">ALAN</a></li> <li>• <a href="#">Number knowledge resources</a></li> <li>• <a href="#">Numeracy resources on the Universal Resources Hub</a></li> <li>• <a href="#">National Numeracy Learning Progression</a></li> <li>• <a href="#">How to – technical guide to using PLAN2</a></li> </ul>

## What does number and place value look like in a K-2 classroom?

These are some examples for the development of number and place value which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022), and key research. For further information refer to the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

### Number knowledge (including place value)

#### Students

- use numbers to help them make sense of and describe the world around them
- connect number words, symbols and quantities (those physically present and imagined)
- make correspondences between collections and express equality or groups
- talk about numbers in terms of their parts (number bonds, part-whole, partitioning numbers into smaller parts)
- explain that all numbers are composed of smaller numbers
- read, represent and record numbers in a variety of ways up to 1000 using objects, pictures, words and numerals to:
  - identify whole quantities as the result of recognising smaller quantities in the range of 1-1000 (for example, uses part-part-whole knowledge of numbers to solve problems)
  - identify zero as both a number (quantity) and a placeholder for reading and writing numbers
- recognise and respond to number words and symbols in all language modes; that is listening, speaking, reading and writing
- read, write, model and describe teen numbers as 1 ten and some more, flexibly renaming numbers. For example, explains 'fourteen can be renamed as 1 ten and 4 more'
- represent the structure of groups of ten in whole numbers, recognising that ten ones is the same as one ten
- compare, model, represent, order and rename 2-digit numbers as counts of tens and ones, renaming numbers flexibly. For example, explains 'thirty-four can be renamed as 3 tens and 4 ones. You can also name it 2 tens and 14 ones, 1 ten and 24 ones or 34 ones'
- apply an understanding of zero when reading numerals that include internal zeros. For example, can read numbers such as 101, 904, 203, and so on
- use knowledge of place value to order numerals up to three-digit numbers state the quantity value of digits in numbers of up to 3-digits (Reasons about quantity)

- use place value to partition and rename three-digit numbers in different ways (Reasons about relations)
- estimate and round whole numbers to the nearest ten or nearest hundred
- instantly recognise (subitise) the number of items in small groups of up to 4 items without counting
- identify standard number configurations such as on a standard dice or dominos or in other arrangements
- determine how many by perceptually subitising
- conceptually subitises by identifying patterns in standard representations, for example, identifies quantities represented in spatial patterns such as dice, dominoes, and so on (see Patterns and algebra section for further information)
- determine how many using conceptual subitising. For example, sees 2 collections of dots – four dots and two dots. Knows 2 more than 4 is 6 so can explain there are 6 dots in total.

## Counting with understanding

### Students

- know the number naming sequences, forwards and backwards from any given number
- understand the stable-order principle that is, to be able to count also means knowing the number names in a repeatable order
- match number words to objects in a one-to-one correspondence
- understand cardinality that is, that the last number counted says, 'how many?'
- recognise conservation and that a count of twelve is still a count of 12 no matter what it looks like
- connect names, symbols and quantities
- count items to determine how many
- make correspondences between collections (Reasons about quantity)
- organise items to keep track of what has been counted and what is yet to be counted
- use the term 'is the same as' to express equality of groups (Reasons about quantity)
- use familiar spatial patterns and structures to support counting. For example, collects items in fives and then counts by fives
- model and describe 'odd' and 'even' numbers using items paired in 2 rows
- count in sequence forwards and backwards by fives and tens on and off the decade up to 100
- estimate the number of items in a group
- identify the number before or after a given number from a range of numerals up to 1000.

# Patterns and algebra

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Patterning is fundamental in all mathematics learning. Amongst other skills, patterning involves the ability to identify and describe attributes of objects and similarities and differences between them (Papic, 2007). Noticing similarities and differences is a powerful way to teach mathematics and it assists in unveiling its structure (Siemon et al, 2019). Developing an intuitive awareness of patterns and structure is fundamental for all students and supports with learning across all mathematical areas and other learning areas. Patterning is integral, for example, to the development of determining ‘how many?’ by subitising and by counting. Patterning also helps us make sense of arithmetic structure, base ten and multiplicative concepts, units of measure, proportional reasoning, data and statistical reasoning.

A mathematical pattern may be described as any replicable regularity, involving number, space or logical relationships. Observing natural patterns in the world around us, creating and describing patterns supports the development of early number concepts inclusive of repeating patterns (for example, ABABAB ...), spatial structural patterns (for example, geometrical shapes), growing patterns (increasing and decreasing), units of measure or transformations (Mulligan 2010, Mulligan and Mitchelmore, 2009).

In the early stages of school, students explore number and pre-algebra concepts by pattern making, and by discussing, generalising and recording their observations. As students become increasingly able to connect patterns with the structure of numbers, they create a foundation for algebraic thinking (that is, thinking about generalised quantities). ‘Generalising patterns is seen as a key to developing mathematical thinking and algebraic understanding’ (Siemon et al, 2015 p.270).

Algebraic thinking is inextricably linked to patterning, and as such, it too is foundational to mathematical thinking. Algebraic thinking provides the language and structure to represent ideas, solve problems, model situations, generalise and prove. Like patterns, algebra is linked with the other strands in the mathematics syllabus and is applicable across a broad range of other learning areas. Algebraic thinking begins in the earliest years of schooling when students describe a repeating pattern as an AB pattern. It can also be found when extend and copy patterns and when they sort and classify objects. We also see algebra in use when we record a situation when I combined 3 donuts with 7 donuts as ‘ $3 + 7 = 10$ ’.

Algebraic understanding supports students with mathematical computation and developing a strong foundational understanding of our numerical system.

Examples of embedded patterning in new Mathematics K-2 syllabus	
Recognises, describes and continues repeating. MAE-FG-01	Copy, continue and create patterns.
Uses the structure of equal groups to solve multiplication problems, and shares or groups to solve division problems. MA1-FG-01	Count in multiples using rhythmic and skip counting.
Applies an understanding of place value and the role of zero to read, write and order two- and three-digit numbers. MA1-RWN-01	Continue and create number patterns.
Recognises, describes and represents shapes including quadrilaterals and other common polygons. MA1-2DS-01	Recognise and classify shapes using obvious features – identify and describe the orientation of shapes using quarter turns.
Uses number bonds and the relationship between addition and subtraction to solve problems involving partitioning. MA1-CSQ-01	Recognise and recall number bonds up to ten – represent equality.



## Syllabus and progression links

Patterning is central to mathematics teaching and learning. Awareness of patterns and knowledge of structure helps learning across all mathematical areas and has a positive influence on mathematical achievement overall to enable a stronger foundation for algebraic thinking.

NSW Syllabus	National Numeracy Learning Progression
<p>All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including <a href="#">Mathematics</a>, <a href="#">Science and technology K-6</a>, <a href="#">Science 7-10</a>, <a href="#">History</a>, <a href="#">Geography</a> and <a href="#">PDHPE</a>. Numeracy is also embedded within <a href="#">Creative arts</a>.</p> <p>Early Stage 1: MAO-WM-01, MAE-FG-01</p> <p>Stage 1: MAO-WM-01, MA1-RWN-01, MA1-CSQ-01, MA1-FG-01, MA1-2DS-01</p> <p>For more information on syllabus connections, see the <a href="#">Making connections in Mathematics</a> document series on the NESA website.</p>	<p>Early Stage 1: NPA1–NPA2</p> <p>Stage 1: NPA2–NPA4</p>

## Further support

Professional learning	Assessment tools and resources
<ul style="list-style-type: none"> <li>• <a href="#">Becoming mathematicians: Exploring patterns</a> (Professional learning video with accompanying resource)</li> <li>• <a href="#">Quality curriculum implementation K-6</a> (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6)</li> <li>• <a href="#">Mathematics K-2 microlearning</a> (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))</li> </ul>	<ul style="list-style-type: none"> <li>• <a href="#">IfSR-Number and place value (IfSR-NP) resources</a></li> <li>• <a href="#">IfSR-NP diagnostic online assessment – ALAN</a></li> <li>• <a href="#">Exploring patterns</a></li> <li>• <a href="#">Talking about patterns and algebra</a></li> <li>• <a href="#">Numeracy resources on the Universal Resources Hub</a></li> <li>• <a href="#">National Numeracy Learning Progression</a></li> <li>• <a href="#">How to – technical guide to using PLAN2</a></li> </ul>

## What do patterns and algebra look like in a K-2 classroom?

These are some examples for the development of patterns and algebra which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022) and key research. For further information see the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

### Students

- explain that a pattern is a mathematical regularity that has a 'core' (repeating unit) that repeats over and over and over
- represent patterns in a variety of ways
- identify, describe and represent a range of patterns such as repeating patterns, growing and shrinking patterns (such as those found in counting), spatial patterns (such as those found in familiar structures such as dominoes and fingers), combinatorial (arithmetical) patterns (such as those found in number bonds to 10), and so on
- create, identify, describe, continue, find missing elements and translate simple and complex repeating patterns
- identify standard and non-standard spatial patterns and structures using finger patterns, dice, ten-frames, dominoes, rekenreks, playing cards and so on
- create, record, describe and continue number patterns that increase or decrease by the same number each time. For example, the pattern of subtracting 1 each time you say the next number word in the backward number word sequence
- determine a missing element in a growing or shrinking pattern, for example, **3, 7, 11, \_\_, 19, 23, 27** and describe how it was determined
- make connections between repeating patterns and counting, for example, a '3' pattern and skip counting by threes
- explain that the symbol = represents a relationship of equivalence
- describe = as 'is equivalent to', 'is equivalent in value to', 'is the same as'
- use reasoning and relational thinking to determine a missing value in an equation (number sentence)  **$7 + ? = 6 + 3$**
- solve additive problems with a missing number by applying the inverse relationship, for example,  **$5 + \_ = 16$  or  $16 - \_ = 5$**
- use derived facts and relationships to solve problems. For example, rethinks  **$135 + 19$  as  $134 + 20$ , or,  $135 + 20 - 1$**
- investigate and generalise the commutative and associative properties to model additive situations. For example, uses a number balance or interlocking cubes to model  **$4 + 5 = 5 + 4$**
- investigate, generate, describe and represent patterns on number lines and number charts.

# Additive thinking

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Additive thinking specifically relates to the construction and representation of number, in particular part-part-whole knowledge which is underpinned by two big ideas: ‘trusting the count’ and ‘place value’.

Part-part-whole knowledge of numbers to 10 is essential to reconfigure numbers in different ways for additive purposes, and this knowledge should be extended upon when decomposing larger numbers. This knowledge allows students to understand numbers as compositions of other numbers, the commutative property of addition and the inverse relationship between addition and subtraction.

Place value knowledge supports the development of mental computation strategies and there should be a focus on the value of working with place value units rather than counts of one. Students should also gain the critical understanding of the place value notion that ‘10 of these is 1 of those’ and extend this understanding and apply it to working with decimals (Siemon et al, 2015, p.270).

The transition from counting by one to more flexible strategies requires students to work on problems in different ways, decomposing, composing, partitioning and/or combining number patterns. Students who can recall number bonds may still rely on strategies like counting on to solve problems. This is why it is important to model the use of number bonds in addition and subtraction problems.

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Choosing a strategy appropriate to the task requires a deep understanding of the operations and a well-developed sense of number. Students should develop a repertoire of meaningful, efficient strategies to support problem solving involving whole numbers, decimals and fractions that they can use confidently to justify their strategy relative to context. Students should be encouraged to engage in discussions that focus on mathematical reasoning to explain and justify the merits of various strategies. When students talk about their solution paths and others’ solution paths, they have opportunities to make sense of ideas and become increasingly able to choose and use additive strategies for different purposes.

## Syllabus and progression links

NSW Syllabus	National Numeracy Learning Progression
<p>All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including <a href="#">Mathematics</a>, <a href="#">Science and technology K-6</a>, <a href="#">Science 7-10</a>, <a href="#">History</a>, <a href="#">Geography</a> and <a href="#">PDHPE</a>. Numeracy is also embedded within <a href="#">Creative arts</a>.</p> <p>Early Stage 1: MAO-WM-01, MAE-RWN-01, MAE-RWN-02, MAE-CSQ-01, MAE-CSQ-02</p> <p>Stage 1: MAO-WM-01, MA1-RWN-01, MA1-RWN-02, MA1-CSQ-01</p> <p>For more information on syllabus connections, see the <a href="#">Making connections in Mathematics</a> document series on the NESA website.</p>	<p>Early Stage 1: NPV1–NPV4, CPr1–CPr5, AdS1–AdS3 and NPA1–NPA2</p> <p>Stage 1: NPV2–NPV6, CPr5–CPr7, AdS1–AdS7 and NPA3–NPA4</p>

## Further support

Professional learning	Assessment tools and resources
<ul style="list-style-type: none"> <li>• <a href="#">Additive strategies</a> – blended professional learning focuses on deepening understanding of the development of additive thinking through integrating practical ideas for classroom application</li> <li>• <a href="#">Becoming mathematicians: flexible additive thinking</a> (Professional learning video with accompanying resource)</li> <li>• <a href="#">Improving reading and numeracy suite</a>:             <ul style="list-style-type: none"> <li>◦ Additive thinking (K-8)</li> </ul> </li> <li>• <a href="#">Quality curriculum implementation K-6</a> (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6)</li> <li>• <a href="#">Mathematics K-2 microlearning</a> (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))</li> </ul>	<ul style="list-style-type: none"> <li>• <a href="#">IfSR-Additive thinking (IfSR-AT) resources</a></li> <li>• <a href="#">IfSR-AT diagnostic online assessment – ALAN</a></li> <li>• <a href="#">Flexible additive strategies – Combinations to 10</a></li> <li>• <a href="#">Flexible additive strategies – 2-digit numbers</a></li> <li>• <a href="#">Flexible additive strategies – 3-digit numbers</a></li> <li>• <a href="#">Flexible additive strategies – Decimals</a></li> <li>• <a href="#">Numeracy resources on the Universal Resources Hub</a></li> <li>• <a href="#">National Numeracy Learning Progression</a></li> <li>• <a href="#">How to – technical guide to using PLAN2</a></li> </ul>

## What does additive thinking look like in a K-2 classroom?

These are some examples for the development of additive thinking which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022) and key research. For further information see the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

### Students

- describe the action of combining, separating and comparing using everyday language, for example, makes, joins, combines with, take away, how many more, altogether, is the same as
- use concrete materials, number lines, visual representations (such as ten-frames) to model and solve simple addition and subtraction problems
- record addition and subtraction in a variety of ways, initially informally using drawings, words and numerals and extending into the use of mathematical symbols
- represent and solve simple addition and subtraction problems using a range of strategies, including counting on, partitioning and rearranging parts
- recognise, recall and record number bonds to 10, and extend this knowledge to assist with combinations for numbers up to 20
- model the commutative property for addition using concrete materials
- recognise that adding or subtracting zero leaves the number (and quantity) unchanged
- relate addition and subtraction facts for numbers to at least 20, using concrete materials to model how addition and subtraction are inverse operations
- use the symbols for plus (+), minus (-) and equals (=)
- recognise that the equals symbol means 'is the same as', rather than an indication to perform an operation and can be used to record equivalent number sentences including recording patterns for individual numbers up to ten by making all possible whole-number combinations
- choose and apply efficient mental strategies for addition and subtraction involving 1- and 2-digit numbers, including counting on or back for small differences, doubles (and near doubles), bridging to 10 and other landmark and benchmark numbers and using place value to partition numbers
- use concrete materials to assist and support conclusions in explanations of how solutions were obtained
- invent ways of recording their actions and engage in discussions about the merits of various strategies in terms of their efficiency communicate and reason to explain how an answer was obtained using terms such as plus, add, minus, the difference between, constant difference, increase, decrease, more than, less than, bridge, number bond, double, equals, strategy, number sentence

- compare their own method of solution with the methods of other students
- explore additive number patterns that increase or decrease
- form multiples of 10 when adding and subtracting 2-digit numbers, and record strategies on an empty number line including bridging 'to' and 'through' multiples of 10
- apply associative and commutative properties and inverse relationship when solving a range of 2-digit addition and subtraction problems to simplify mental computation
- represent a wide range of additive problem situations involving 2-digit numbers using appropriate addition and subtraction number sentences.

# Multiplicative thinking

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According to Siemon et al. (2019) multiplicative thinking is characterised by:

- a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole numbers, decimals, common fractions, ratio, and percentages)
- an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion
- the means to communicate this effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms).

In short, multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts (Siemon and Breed, 2005).

In order to think multiplicatively and work with multiplication and division formally, students need to be able to count large collections efficiently and also recognise the numbers 2-10 as composite units. Teachers can assist students to notice different counting strategies and patterns through regular subitising activities and opportunities to count large collections of items. Furthermore, students need to develop efficient, flexible mental strategies and be able to work with; part-part-whole knowledge to 10 (that is, recognise numbers to 10 in terms of their parts); mental strategies for addition and subtraction with numbers to at least 20 (for example, doubles, near doubles); place value ideas flexibly (including renaming numbers) and sharing concrete materials equally.

The transition from additive to multiplicative thinking and strategies requires a significant shift in thinking, from counting and additive processes (counting equal groups and repeated addition) to utilising the ‘for each’ and ‘times as many’ ideas for multiplication. The ability to mentally coordinate 2 composite units supports the critical shift in student thinking from counting groups to recognising and operating with factors.

Array-based strategies support the shift from an additive groups of model to a factor-factor-product model (multiplicative) which is needed to support fraction representation, the multiplication and division of larger whole numbers, fractions and decimals, and algebra. Arrays can be used for building multiplication facts in a meaningful way. Before developing multiplicative automaticity, children must understand how these facts are derived. Siemon et al. (2019) explain that ‘Array and region representations provide a basis for a more generalised understanding of multiplication and division that can be used to support the multiplication of multi-digits, fractions and, ultimately, algebraic expressions’ (p.387).

Multiplicative structures and situations include equal groups ideas for both multiplication and division context, partition, ‘times as many’ (multiplicative comparison), arrays (extending into area and region ideas) and the ‘for each’ idea (or Cartesian product idea).

The capacity to think multiplicatively underpins proportional thinking and provides the basis for working with the relationships between quantities for concepts such as fractions, decimals, percentages, rates and ratios.

Fractions are introduced through equal-sharing problems. As Empson and Levi (2011) stated, 'If models do not draw on children's formative experiences of sharing and partitioning, then they are likely to prevent teachers from cultivating the natural insights about quantities that young children have'. Students can later connect the words and symbols for fractions to the parts created in the process of solving the problems. The concept of a fraction is then based on the process of division, rather than counting parts. This lays the foundations for fractions as indicating division and fractions as numbers.

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## Syllabus and progression links

NSW Syllabus	National Literacy Learning Progression
<p>All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including <u>Mathematics</u>, <u>Science and technology K-6</u>, <u>Science 7-10</u>, <u>History</u>, <u>Geography</u> and <u>PDHPE</u>. Numeracy is also embedded within <u>Creative arts</u>.</p> <p>Early Stage 1: MAO-WM-01, MAE-FG-01, MAE-FG-02, MAE-GM-03</p> <p>Stage 1: MAO-WM-01, MA1-FG-01, MA1-GM-03</p> <p>For more information on syllabus connections, see the <u>Making connections in Mathematics</u> document series on the NESA website.</p>	<p>Early Stage 1: MuS1–MuS2, InF1–InF2 and NPA1–NPA2</p> <p>Stage 1: CPr6, MuS2–MuS6, InF1–InF3 and NPA3–NPA4</p>



## Further support

Professional learning	Assessment tools and resources
<ul style="list-style-type: none"> <li>• <a href="#">Multiplicative strategies: blended learning</a> (a combination of self-paced modules and facilitator-led, live sessions via Microsoft Teams)</li> <li>• <a href="#">Becoming mathematicians: Flexible multiplicative thinking</a></li> <li>• <a href="#">Improving reading and numeracy suite</a>:               <ul style="list-style-type: none"> <li>◦ Multiplicative thinking (K-8)</li> </ul> </li> <li>• <a href="#">Quality curriculum implementation K-6</a> (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6)</li> <li>• <a href="#">Mathematics K-2 microlearning</a> (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))</li> </ul>	<ul style="list-style-type: none"> <li>• <a href="#">IfSR-Multiplicative thinking (IfSR-MT) resources</a></li> <li>• <a href="#">IfSR-MT diagnostic online assessment – ALAN</a></li> <li>• <a href="#">Strategies for sharing and forming groups</a></li> <li>• <a href="#">Multiplicative thinking with single digit numbers</a></li> <li>• <a href="#">Multiplicative thinking with multi digit numbers</a></li> <li>• <a href="#">Flexible strategies with rational numbers</a></li> <li>• <a href="#">Multiplicative strategies webpage</a></li> <li>• <a href="#">Numeracy resources on the Universal Resources Hub</a></li> <li>• <a href="#">National Numeracy Learning Progression</a></li> <li>• <a href="#">How to – technical guide to using PLAN2</a></li> </ul>

## What does multiplicative thinking look like in a K-2 classroom?

These are some examples for the development of multiplicative thinking which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022) and key research. For further information see the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

### Students

- investigate, model (using concrete materials), record and describe equal groups and sharing in response to authentic situations
- record grouping and sharing informally using pictures, words and numerals
- link multiplication and division by dividing a quantity into equal groups and then reversing the process by combining the groups to form one collection again, thus linking multiplication and division
- communicate about and distinguish between the ‘how many’ (number of groups) and the ‘how much’ (number in each group) when describing collections of objects
- trust the count and sees equal groups as a composite unit
- skip count to determine how many in a collection (by twos, threes, fives and tens)

- model multiplication as repeated addition (and subtraction) and use an empty number line to record their thinking
- explore the application of repeated addition and repeated subtraction in practical situations
- model and use arrays (of equal columns and rows) to work with numbers as composite units and to compose and decompose numbers flexibly
- describe the part left over when a collection cannot be distributed equally using the given group/row/column size, including reasoning using array models
- model the commutative property of multiplication, including by rotating arrays 90 degrees
- apply doubling (and halving strategies) when working with whole numbers and fractions –for example, when sharing a collection of objects into 2 groups, students may describe the number of objects in each group as being one-half of the whole collection
- use concrete materials to model a half of a linear arrangement of a collection, then a quarter and an eighth and show the relation between the part and the whole
- subdivide lengths (continuous model) to find halves, quarters and eighths, identifying the relationship of the parts to the whole length
- establish an understanding of the ‘for each’ idea when solving simple, multiplicative problems (for example, for each party bag there are six lollies) and extending this to the multiplicative structure of the base ten place value system (for example, for each hundred, there are 10 tens) communicate and reason to explain how an answer was obtained and compare their own method of solution with the methods of other students.

# Proportional thinking

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Proportional thinking is a complex form of reasoning that builds upon a number of interconnected ideas over a long period of time (Siemon et al. 2019). This means that teachers of Kindergarten students can support the foundations for proportional thinking. Proportional thinking requires skills in thinking multiplicatively and involves measures, rates and/or ratios expressed in terms of natural numbers, rational numbers, and/or integers. For example,  $\frac{2}{3} \times \$24$  as **two-thirds of \$24**, or  $3.5 \times 68$  as **three and a half times 68**, (Siemon et al., 2019). Students need to understand and work with percentages, fractions, rates, ratio and represent the proportional relationship between quantities.

Students apply their number skills to a variety of situations, including financial situations and practical problems, developing a range of life skills important for numeracy. Ratios and rates underpin proportional reasoning needed for problem solving and the development of concepts and skills in other aspects of mathematics, such as trigonometry, similarity and gradient.

Proportional reasoning also includes tasks involving a comparison of different rates, for example, if one dog grows from 5 kilograms to 8 kilograms and another dog grows from 3 kilograms to 6 kilograms, which dog grew more compared to its original weight? Learning to reason using proportion is a complex process that develops over an extended period (ACARA, 2020).

## What does proportional thinking look like in a K-2 classroom?

### Students

In the early years, place value, multiplication, division and fractions introduce ideas of proportionality (Van de Walle, 2015). Students begin reasoning proportionally as they divide quantities using the ideas of ‘fair’ and ‘the same’ in developing a sense of equality and compare the relationship between quantities for example ‘twice as big’ and ‘half as much’ (Department of Education and Training Victoria).

Students need to be able to work meaningfully with a wide range of numbers and have access to multiplicative thinking, for example in terms of coordinating composite units and using the ‘for each’ idea.

For further information, see **Multiplicative thinking** K-2 (pp. 31-34) and **Proportional thinking** in the Numeracy guide Years 3 to 8 (pp. 37-39).

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